Hypernuclear structure and the photoproduction

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HaPhys Workshop 2014
Soongsil University, Seoul
March 3 - 4, 2014
Recollecting: APCTP Workshop on Strangeness Nuclear Physics (1999)
Seoul Nat. Univ., successfully organized by Il-T. Cheon (Yonsei), S.-W.Hong (Sunkyunkan) and T.Motoba (Osaka E-C.)
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   4-4. ($\gamma$,K$^+$) reaction to produce sd-shell and heavier systems
5. Concluding remarks
1. Introduction

(From H. Tamura)
More bound states: Hypernuclei provide us with new opportunities to study V(Y-N) and dynamics of new baryon many-body systems.
Hypernuclear Chart

Taken from N. Nakamura, based on O. Hashimoto and H. Tamura
Progress in disclosing $\Lambda^9\text{Be}$ level structure

high precision $\gamma$-rays (Tamura et al)

Microscopic Cluster model,
Motoba, Bando, Ikeda, PTP79, 189 (1983)
2. Elementary hyperon production processes and the characteristics

Three factors:
1. Hyperon recoil momentum
2. Selectivity due to process
3. Cross section
(1) Hyperon recoil momentum $q_Y$ as a function of projectile momentum

![Graph showing the dependence of hyperon recoil momentum on projectile momentum. The graph includes various reactions and their corresponding recoil momentum values at different projectile momenta.](image-url)
Selectivity of hypernuclear production

(K⁻,π⁻) at p=0.8 GeV/c:
- Recoilless production of Λ
- Substitutional states with ΔL=0,1

(π⁺,K⁺) at p=1.05 GeV/c:
- Natural parity high-spin stretched states

(γ,K⁺) at p=1.3 GeV/c:
- Unnatural parity high-spin states

$q_\Lambda=350-420$ MeV/c at $E_\gamma=1.3$ GeV
(3) Cross section and polarization

\[ \pi^+ n \rightarrow \Lambda K^+ \]

Elementary process

Cf. K- beam at J-PARC
3. Production of Hypernuclei: Reactions on the nuclear target

Four factors:

1. PW vs. DW (Meson DW effects)
2. Microscopic treatment with elem. amplitudes,
3. Nuclear target has its own structure
4. Nuclear core excitation effects on hypernuclear production rates

\[ {}^A Z(J_i T_i \tau_i)(K^-, \pi^-)^A Z(J_f T_f \tau_f) \]

\((\pi^+, \text{K}+)\)
\((\gamma, \text{K}+)\)
\((\text{K}^-, \text{K}^+)\)
Choose $^{19}$F(1/2$^+$) target for demonstration

Shell-model configuration

$^{16}$O

$^{19}$F

neutron

proton

$\Lambda$ (DDHF)
As a “closed core (\(^{18}\text{O}\))” + \(\Lambda\), cf. SO-splitting(0p)=152+/−54 keV(C13)
A BRIEF LOOK INTO REACTION THEORIES

(A) FACTORIZED VS. (B) MICROSCOPIC

(A) Factorized DWIA treatment

by Huefner-Lee-Weidenmueller, NPA234, 429 (1974)

\[
\frac{d\sigma(\theta)}{d\Omega_L}^{(K^-,\pi^-)} = \alpha \left( \frac{d\bar{\sigma}(\theta)}{d\Omega_L} \right)^{K^-n\rightarrow\Lambda\pi^-} N_{\text{eff}}^{(K^-,\pi^-)}(i_f; \theta)
\]

\(\alpha = \) kinematical factor for A-body to 2-body transformation,

\(N_{\text{eff}} = \) Effective neutron number:

\[
N_{\text{eff}}^{(K^-,\pi^-)}(i_f; \theta) = \frac{1}{[J_i]} \sum_{M_iM_f} \left| \left< J_f M_f T_f \tau_f \middle| \int dr \chi_{\pi^-}^{(-)*} \left( \frac{M_A}{M_H} r \right) \chi_{\pi^+}^{(+)}(k_K, r) \right| \right|^2 
\]

\[
\times \sum_{j=1}^{A} U_-(j) \delta \left( r - \frac{M_c}{M_A} r_j \right) \left| J_i M_i T_i \tau_i \right|^2.
\]
(A-1) Meson waves by the Eikonal approximation

\[ N_{\text{eff}}(Z_{\text{eff}}) = \int \rho_{n(p)}(r) \cdot \exp \left[ -\bar{\sigma}_{aN} \int_{-\infty}^{z} \rho(x, y, z') dz' \right. \]
\[ \left. -\bar{\sigma}_{bN} \int_{z}^{\infty} \rho(x, y, z') dz' \right] d^3r \]

Applicable to forward scattering, 10-20% error of K-G DW (Auerbach et al (1983)).

(A-2) Meson DW with the Klein-Gordon solutions (Ours)

\[ N_{\text{eff}} = \frac{A}{2J + 1} \sum_{M_iM_f} \left| \int \Psi_f^*(A^A Z) \hat{O}(r) \Phi_i(A^A Z) \prod_{i=1}^{A} dr_i \right|^2, \]

\[ \hat{O}_{ab}(\theta; r) = \int d^3r \ \chi_{p_b}^{(-)*}(r) \cdot \chi_{p_a}^{(+)}(r) \sum_{i=1}^{A} \hat{X}(i) \delta(r - r_i) \]

\[ \chi_{p_b}^{(-)*}(r) \cdot \chi_{p_a}^{(+)}(r) = \sum_{\kappa} \sqrt{4\pi[\kappa]} \ i^\kappa \ \hat{\tilde{j}}_\kappa(p_b, p_a, \theta; r) Y_\kappa(\hat{r}_{ab}). \]
PW vs. DW

(1) In a typical \((\pi^+, K^+)\):
\[ N_{\text{eff}} = 0.184 \text{ (PW)} \]
\[ 0.030 \text{ (DW)} \]

(2) XS to low-J states are much more reduced, resulting in the sharper peaks

(3) Interesting DW effect (mechanism)
Low-L partial waves are more reduced by absorption effect, leading to well-separated high-L series of peaks.
(B) Microscopic treatment based on the elementary transition amplitudes

\[
\frac{d\sigma(\theta_L)}{d\Omega_L} = \gamma \cdot \frac{(2\pi)^4 p_K^2 E_\pi E_K E_H}{p_\pi \{p_K(E_H + E_K) - p_\pi E_K \cos \theta_L\}} |T_{if}^L|^2,
\]

\[
|T_{if}^L|^2 = \sum_{M_f} R(if; M_f),
\]

\[
R(if; M_f) = \frac{1}{[J_i]} \sum_{M_i} \left| \langle J_f M_f | \int d^3 r \chi^{(-)}(p_K; r)^* \cdot \chi^{(+)}(p_\pi; r) \right| \times \sum_{k=1}^{A} U_-(k) \delta(r - r_k) \cdot \lambda [f + ig(\sigma_k \cdot \hat{n})] |J_i M_i\rangle \right| ^2,
\]

Elementary amplitude \( N \rightarrow Y \)

\( f = \text{spin-nonflip}, \ g = \text{spin-flip}, \ \sigma = \text{baryon spin} \)
$R(i,f;M)$ is expressed with three kinds of the reduced effective numbers

$$R(i,f;M_f) = \lambda^2 \{ |f|^2 \rho^{ff}(i,f;M_f) + |g|^2 \rho^{gg}(i,f;M_f) + 2 \text{Im}[fg^*] \rho^{fg}(i,f;M_f) \}$$

Magnetic subspace population $P(i,f:M)$ is defined by

$$P(i,f;M_f) = \frac{R(i,f;M_f)}{\sum_{M_f} R(i,f;M_f)}$$

Polarization of Hypernuclear state $|J_f>$ is calculated by

$$\mathcal{P}(J_f) = \sum_{M_f} M_f P(i,f;M_f) / J_f.$$
4. Detailed hypernuclear structure
Calculations vs. Reaction spectroscopy

Extend the calculation framework to predict/explain experimental strength functions: (four examples selected)

4.1 Microscopic cluster model applied to $^9\Lambda$Be
4.2 Microscopic cluster model applied to $^7\Lambda$Li
4.3 Shell-model analyses of $^89\Lambda$Y(\pi+,K+) $^9\Lambda$Y
4.4 Shell-model prediction for $^{28}\Lambda$Si(\gamma,K+) $^{28}\Lambda$Al
4.1 Microscopic cluster model applied to $^9\Lambda$Be
Microscopic Cluster Model of $^{9}_{\Lambda}\text{Be} = (\alpha + 3N + N) + \Lambda$

$\Phi^\Lambda_J(9\text{Be}) = \sum_{C,D} \omega^\Lambda_C(D) \left[ \Phi^T_{\Lambda,LSI_i} (8\text{Be}) \right] \times \frac{\Phi^\Lambda_{\Lambda}(R;D) \chi_\Lambda(\bar{R})}{G_c \text{ basis}}$

$C = (LSI_i, \Lambda)_J$

Hamiltonian: $H = h^{\text{OCM}}_{\text{I}} (8\text{Be}) + T_\Lambda + V_{AN}$

$V_{AN} = \sum_{i=1}^{8} U_{AN}(\Lambda, i), \quad U_{AN}(\Lambda) = -38.19 \exp[-1.43/1.035^2]$}

\[ \Sigma \left\{ T^\Lambda_{\Lambda} \right\}_{C_2,D_2} \left( E_J - E_{c_J} \right) \left\{ N^\Lambda_{\Lambda} \right\}_{C_2,D_2} \delta_{c_2} + U_J(c_1,d_1,c_2,d_2) \right] \omega^\Lambda_{C_2}(R_2) = 0 \]

where

\[ \left\{ T^\Lambda_{\Lambda} \right\}_{C_2,D_2} = \langle \Phi^\Lambda_{\Lambda}(R;D) | \left\{ T^\Lambda_{\Lambda} \right\} | \Phi^\Lambda_{\Lambda}(R;D) \rangle \]

\[ U_J(c_1,d_1,c_2,d_2) = \langle \Phi^\Lambda_{\Lambda}(R;D_1) | \tilde{U}_J(R;C_1,C_2) | \Phi^\Lambda_{\Lambda}(R;D_2) \rangle \]

\[ \tilde{U}_J(R;C_1,C_2) = \langle [\Phi^T_{\Lambda,LSI_i} (8\text{Be}) \chi_\Lambda]_J | V_{AN} | [\Phi^T_{\Lambda,LSI_i} (8\text{Be}) \chi_\Lambda]_J \rangle \]

\[ \varepsilon_c = \langle \Phi^T_{\Lambda,LSI_i} (8\text{Be}) | h^{\text{OCM}} (8\text{Be}) | \Phi^T_{\Lambda,LSI_i} (8\text{Be}) \rangle \]

USE SOLUTIONS FOR $^8\text{Be}$
All the existing exp.data can be explained.
4.2 Shrinkage effect due to the $\Lambda$ participation to the nucleus
Shrinkage due to $\Lambda$ participation
( "glue-like role" of $\Lambda$ )

predicted by Microscopic $\alpha+d+\Lambda$ model

*Microscopic Λ5He+p+n model to see free p and n dynamics.*

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**FIG. 3.** Jacobian coordinates of the three rearrangement channels of the "core"+N+N model. Here, "core" is α for the A = 6 nuclei and \(^5\)\(^\Lambda\)He for the A = 7 hypernuclei.
Contraction of $R_{\text{core}}-(pn)$ without changing p-n distribution (Hiyama et al., 1998)

**Fig. 4.** (a) The $n-p$ relative density distribution $\rho(r_{n-p})$ defined by Eq.(4.6) multiplied by $r_{n-p}^2$. (b) The $(np)$ pair c.m. density distribution $\rho(R_{\text{core}}-(np))$ defined by Eq.(4.7) multiplied by $R_{\text{core}}^2$. Both are for the ground states of $^6\text{Li}$ and $^7\Lambda\text{Li}$. 

\[
\rho(R) = \int |\psi|^2 \, dr \, dr' / 4\pi
\]
B(\(^6\text{Li}, E2; 3^+ \rightarrow 1^+)\) vs. B(\(^\Lambda^7\text{Li}, E2; 5/2^+ \rightarrow 1/2^+)\)
Shrinkage: glue-like role of Λ confirmed

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</thead>
<tbody>
<tr>
<td>B(M1:3/2+ → 1/2+)</td>
<td>0.364</td>
<td>0.352</td>
<td>0.322</td>
<td>-</td>
</tr>
<tr>
<td>B(E2:5/2+ → 1/2+)</td>
<td>8.6</td>
<td>2.46</td>
<td>2.42</td>
<td>4.1+-1.1</td>
</tr>
<tr>
<td>*eff.chrg: δe=0.15e</td>
<td>8.6</td>
<td>(4.16*)</td>
<td>(4.09*)</td>
<td></td>
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<tr>
<td>B(E2:5/2+ → 3/2+)</td>
<td>3.1</td>
<td>0.40</td>
<td>0.74</td>
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<tr>
<td>B(E2:5/2+ → all)/B(E2)c</td>
<td>1.0 assumed</td>
<td>0.44</td>
<td>0.33</td>
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<tr>
<td>ΓB</td>
<td>0.49</td>
<td>0.32</td>
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<tr>
<td>R_{cd}^{7Li}/R_{αd}^{6Li}</td>
<td>0.83</td>
<td>0.75</td>
<td>0.87</td>
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<tr>
<td>Lifetime(5/2+)_ps</td>
<td>6.56</td>
<td>6.67</td>
<td>5.2+-1.4</td>
<td></td>
</tr>
</tbody>
</table>
4.3 Shell-model analyses of $^{89}\Lambda Y(\pi^+, K^+)$ $^{89}\Lambda Y$
Probe deep-lying $\Lambda$ single-particle states
\[ F(q) = \frac{\Gamma(J + 3/2)^2(2Z)^J e^{-Z}}{[(2J + 1)!!]^2 \Gamma(l_n + 3/2) \Gamma(l_\Lambda + 3/2)} \]

For 
\[ [(0|n j_n)^{-1}(0|\Lambda j_\Lambda)] J \]

F.F becomes maximum at 
\[ J = (bq)^2 / 2 \]
How to understand $\Lambda^{89}\text{Y}$ data (Hotchi et al, PRC64 (2001)) in view of CAL (Motoba et al, 1988)
If we assume that each doublet corresponds to $j_>$ and $j_<$, two serious problems arise:

1. Energy splittings are not proportional to $(2\ell + 1)$.

<table>
<thead>
<tr>
<th>Peaks</th>
<th>$E_\Lambda$ (MeV)</th>
<th>$\Delta E_\Lambda$</th>
<th>$\Delta E_\Lambda$ ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>EXP</td>
<td>HO</td>
</tr>
<tr>
<td>$\ell=0$</td>
<td>-23.11</td>
<td></td>
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<tr>
<td>$\ell=1$ L</td>
<td>-17.10</td>
<td>1.37 (1.0)</td>
<td>(1.0)</td>
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<tr>
<td>R</td>
<td>-15.73</td>
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<tr>
<td>$\ell=2$ L</td>
<td>-10.32</td>
<td>1.63 (1.19)</td>
<td>(1.67)</td>
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<tr>
<td>R</td>
<td>-8.69</td>
<td></td>
<td></td>
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<tr>
<td>$\ell=3$ L</td>
<td>-3.13</td>
<td>1.70 (1.24)</td>
<td>(2.33)</td>
</tr>
<tr>
<td>R</td>
<td>-1.43</td>
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</tbody>
</table>
Models for Structure of $^{89}_{\Lambda}Y$:

(1) The simplest model with 1-hole core (assume a $^{90}$Zr target)

(2) Role of an odd proton introduced (single $1p-1h$ core): 2 levels in $^{88}$Y.
Models for Structure of $^{89}_{\Lambda}$Y:

(3) Many $[1p-1h]_{J_c}$ multiplets of the $^{88}$Y core excitation due to $V_{NN}$.

$\Lambda$ orbits (DDHF: $\delta=0$)

\[ [(j_p j_n^{-1})_{J_c} j^\Lambda] J_{tot} \]
3L and 3R peaks as a function of $\delta$
$^{89}\text{Y}(\pi^+, K^+) ^{89}_\Lambda \text{Y}$

$p_\pi = 1.05$ GeV/c

$\theta_K = 0$

DWIA

$\delta(0f) = 0.2$ MeV

$d^2\sigma/d\Omega dE$ (nb/sr/MeV)

Hypernuclear Excitation Energy $E_\Lambda$ (MeV)
Spin–spin and spin–orbit splitting at $A=90$
Nijmegen B–B interaction model improved by taking account of hypernuclear data
4.4 Theoretical prediction vs. Jlab experiment
\[ ^{28}\text{Si}(\gamma, K^+) \Lambda^{28}\text{Al} \]
and other predictions
Cf. energy resolution
\((K^-, \pi^-)\)
\(\Gamma = 3\text{-}4 \text{ MeV}\)

\((\pi^+, K^+)\)
played a great role of
exciting high-spin series
\(\Gamma = 1.5 \text{ MeV (best)}\)

\((e, e'K^+), (\gamma, K^+)\)

Motoba, Sotona, Itonaga,
updated w/NSC97f.

JLab Exp’t : \(\Gamma = 0.5 \text{ MeV}\)
Lab $d\sigma/d\Omega$ for photoproduction $(2\text{Lab})$

$$
\frac{d\sigma}{d\Omega}_{2\text{Lab}} = \frac{(2\pi)^4 p^2 E_K E_\gamma E_\lambda}{k\{p(E_\lambda + E_K) - kE_K\cos\theta_L}\} \left| \langle k-p, p|t|k, 0\rangle_L \right|^2 ,
$$

$$
\langle k-p, p|t|k, 0\rangle_L = a_1(\sigma \cdot \epsilon) + a_2(\sigma \cdot \hat{k})(\hat{p} \cdot \epsilon) + a_3(\sigma \cdot \hat{p})(\hat{p} \cdot \epsilon) + a_4((\hat{k} \times \hat{p}) \cdot \epsilon).
$$

Spin non-flip term

$$
\langle k-p, p|t|k, 0\rangle_L = \epsilon_0(f_0 + g_0\sigma_0) + \epsilon_x(g_1\sigma_1 + g_{-1}\sigma_{-1})
$$

with definitions of the coefficients:

$$
f_0 = a_4 \sin\theta_L ,
$$

$$
g_0 = a_1 ,
$$

$$
g_{\pm1} = \frac{1}{\sqrt{2}}\{\mp(a_1 + a_3\sin^2\theta_L) - is\sin\theta_L(a_2 + a_3\cos\theta_L)\}.
$$

Spin-flip interaction are dominant
4.4a Single-$j$ model for the $^{28}$Si target

A typical example of medium-heavy target: $^{28}$Si: $(d_{5/2})^6$

to show characteristics of the (γ,K$^+$) reaction with DDHF w.f.

( Spin-orbit splitting:
consistent with $^7$Li, $^9$Be, $^{13}$C, $^{89}$Y )
Theor. x-section for \((d_{5/2})^6 (\gamma,K^+)\) \([j_h-j_\Lambda]J\)

<table>
<thead>
<tr>
<th>DWIA</th>
<th>(s_{1/2L}) (-16.92)</th>
<th>(p_{3/2L}) (-8.40)</th>
<th>(p_{1/2L}) (-8.40)</th>
<th>(1s_{1/2L}) (0.32)</th>
<th>(d_{5/2L}) (0.69)</th>
<th>(d_{3/2L}) (0.69)</th>
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<td>(2^+)</td>
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<td>(3^+)</td>
<td>63.8</td>
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<td>(4^+)</td>
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<td>(g.s.)</td>
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<td>(0^+)</td>
<td>1.0</td>
<td>0.0</td>
<td>1+ 26.0</td>
<td>1+ 8.9</td>
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<td>(1^+)</td>
<td>7.1</td>
<td>19.4</td>
<td>2+ 2.2</td>
<td>2+ 34.9</td>
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<td>(2^+)</td>
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<td>76.2</td>
<td>3+ 4.6</td>
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4-4b. Realistic prediction for $^{28}\text{Si} (\gamma,K^+) \rightarrow ^{28}\text{Al}$

By fully taking account of
-- full $p(sd)^6.n(sd)^6$ configurations,
-- fragmentations when a proton is converted,
-- $^{27}\text{Al}$ core nuclear excitation
-- $K^+$ wave distortion effects

→ Comparison with the $^{28}\text{Si} (e,e'K^+) \exp.$
proton-state **fragmentations** should be taken into account **to be realistic**
Proton pickup from $^{28}\text{Si}(0^+): (sd)^6 = (d_{5/2})^{4.1}(1s_{1/2})^{0.9}(d_{3/2})^{1.0}$
Peaks can be classified by the characters

\[ ^{28}\text{Si}(\gamma,\text{K}^{+})^{28}_\Lambda\text{Al} \quad E_\gamma = 1.3 \text{ GeV} \quad \theta_K = 3^\circ \]

![Graph showing peaks and their classifications.]

\[ \text{d}^2\sigma/\text{d}\Omega\text{d}E \quad (\text{nb/sr/MeV}) \]

Hypernuclear Energy \( E_\Lambda \) (MeV)

Major peak series: \[ ^{27}\text{Al}(5/2^+)_1 \times j^\Lambda \]\n
with \( j^\Lambda = s, p, d, \ldots \)
Great progress when compared with
cf. $^{28}\text{Si}(\pi^+,K^+)$
Seems promising, (waiting for the finalization of exp. analysis)
$^{40}$Ca (LS-closed shell case):

high-spin states with natural-parity ($2^+, 3^-, 4^+$)

$^{40}$Ca($\gamma, K^+)^{40}$K  $E_\gamma = 1.3$ GeV  $\theta_K = 3^\circ$

Major peak series: $[^{39}K(d_{3/2}^{-1}; gs) \times j^\Lambda]_J$ with $j^\Lambda = s, p, d, f, ...$
Elem. ampl.  Theor. prediction vs. \((e,e'K^+)\) exp.

Motoba. Sotona, Itonaga,  
updated w/NSC97f.

--------------------------------------- Sotona’s Calc.----→  
Hall C (up) T. Miyoshi et al.  
*P.R.L.* **90** (2003) 232502. \(\Gamma = 0.75\) MeV  
Hall A (bottom), J.J. LeRose et al.  
*N.P. A804* (2008) 116. \(\Gamma = 0.67\) MeV
A possible test of models at forward angles

DWIA calculation of the cross sections for the electroproduction of hypernuclei


elementary amplitude: Saclay-Lyon A

JLab data E94-107: \( W = 2.2 \text{ GeV}, Q^2 = 0.07 \text{ (GeV/c)}^2 \)

M. Iodice, ... M. Sotona, ...

<table>
<thead>
<tr>
<th>( E_x ) (MeV)</th>
<th>cross sections (nb/\text{st}^2/\text{GeV})</th>
<th>Exp.</th>
<th>Theor.</th>
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<td>0.0</td>
<td>4.48±0.29</td>
<td>4.68</td>
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<tr>
<td>2.65</td>
<td>0.75±0.16</td>
<td>1.54</td>
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<td>5.92</td>
<td>0.45±0.13</td>
<td>0.76</td>
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<tr>
<td>10.93</td>
<td>3.42±0.50</td>
<td>3.98</td>
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</tbody>
</table>

Possible test of $\gamma p\rightarrow\Lambda K$ ampl.

Comparison of isobar and Regge-plus-resonance models

**H2**: isobar model with hadronic f.f.; fit to CLAS data; nucleon resonances: $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $D_{13}(1895)$; hyperon resonances: $S_{01}(1670)$, $S_{01}(1800)$

**RPR**: fit to CLAS and LEPS data (cross sections) with resonances $S_{11}(1535)$, $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $D_{13}(1895)$; multidipole-Gauss hadronic f.f.;

motivated by RPR-2011B [Lesley De Cruz, PhD thesis, Ghent University, 2011]

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**Figure**: Graph showing differential cross section $d\sigma/d\Omega$ for the reaction $p(\gamma, K^+)\Lambda$ at $E_{\gamma}^{lab} = 2.2$ GeV and $E_{\gamma}^{lab} = 1.3$ GeV. The graph compares different models and data sets, including CLAS, SAPHIR03, and LEPS.

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(From P. Bydzovsky)
6. Concluding remarks

1) discussed interesting topics selected in hypernuclear spectroscopy, by focusing the relation with production processes.

2) emphasized novel aspects of many-baryon systems with hyperon(s).

3) emphasized importance of producing medium and heavy hypernuclei with good energy resolution.

4) skipped the details of YN interactions, those experiments will provide with interesting dynamical structures together with YN interaction properties.
Life of strange many-body systems

Life of the Hypernucleus
from birth to "death"

\[ \Sigma N \rightarrow \Lambda N \rightarrow \pi NN \]

- \( \Lambda, \Sigma \)
- \( p, n, \alpha \) (\( k, \pi \alpha \)) (\( \pi, K \alpha \))
- \( p, n, \alpha \)
- \( \gamma \) E1
- \( \gamma \) E2, M1
- \( \gamma \) decay
- \( \pi \)-decay

Hy (K, \( \pi \gamma \))

N

Hy

structure

nonmesonic decay
Thank you for your attention.